

Fairness in Academic Course Timetabling*

Moritz Muehlenthaler Rolf Wanka

Department of Computer Science
University of Erlangen-Nuremberg, Germany
{moritz.muehlenthaler,rwanka}@cs.fau.de

Abstract

We consider the problem of creating *fair* course timetables in the setting of a university. Our motivation is to improve the overall satisfaction of individuals concerned (students, teachers, etc.) by providing a fair timetable to them. The central idea is that undesirable arrangements in the course timetable, i. e., violations of soft constraints, should be distributed in a fair way among the individuals. We propose two formulations for the fair course timetabling problem that are based on max-min fairness and Jain’s fairness index, respectively. Furthermore, we present and experimentally evaluate an optimization algorithm based on simulated annealing for solving max-min fair course timetabling problems. The new contribution is concerned with measuring the energy difference between two timetables, i. e., how much worse a timetable is compared to another timetable with respect to max-min fairness. We introduce three different energy difference measures and evaluate their impact on the overall algorithm performance. The second proposed problem formulation focuses on the tradeoff between fairness and the total amount of soft constraint violations. Our experimental evaluation shows that the known best solutions to the ITC2007 curriculum-based course timetabling instances are quite fair with respect to Jain’s fairness index. However, the experiments also show that the fairness can be improved further for only a rather small increase in the total amount of soft constraint violations.

1 Introduction

The university course timetabling problem (UCTP) captures the task of assigning courses to a limited number of resources (rooms and timeslots) in the setting of a university. In this work, we consider the problem of creating fair course timetables in the context of a particular variant of the

*Research funded in parts by the School of Engineering of the University of Erlangen-Nuremberg. A preliminary version appeared in: Proc. 9th Int. Conf. on the Practice and Theory of Automated Timetabling (PATAT), 2012, pp. 114–130.

UCTP, the curriculum-based course timetabling (CB-CTT) problem proposed in [DGMS07]. The CB-CTT formulation features various hard and soft constraints which model typical real-world requirements. The hard constraints are basic requirements, for example, no two lectures may be held in the same room at the same time. The feasible solutions to a CB-CTT instance are the timetables satisfying all hard constraints. Soft constraints characterize properties of a course schedule which are undesirable for the stakeholders. The quality of a feasible timetable is determined by the extent to which the soft constraints are violated. A soft constraint violation results in a penalty, and the task is to find a feasible timetable such that the total penalty is minimal. Situations may arise however, in which a large part of penalty hits only a small group of the stakeholders, who would thus receive a poor timetable in comparison to others. In other words, a timetable may be unfair due to an unequal distribution of penalty. In this work, we apply fairness criteria to the CB-CTT problem. Our goal is to achieve a balance of interests between the stakeholders by imposing fairness conditions on the distribution of penalty and thus, to improve the overall stakeholder satisfaction.

In general, fairness is of interest whenever scarce resources are allocated to stakeholders with demands. In economics for example, the distribution of wealth and income and how to measure inequality of resource distributions is of major concern, see for example [FS06] and [SF97]. In computer science, fairness is a central theme in design and analysis of communication protocols (see for instance [BFCYZ02, BG92, BFT11, JCH84, KMT98, KRT01, OW04, SB08]). In operations research, fairness criteria have been applied for example to the aircraft landing problem by [SK08]. In the literature, there is a wealth of different definitions of how to determine the fairness of a given resource distribution. For instance, we may consider the total amount of allocated resources, the outcome for the worst-off stakeholder, the deviation from the mean allocation, and so forth. We propose two fair variants of the CB-CTT problem, which differ with respect to the underlying notion of fairness. Our first problem formulation, MMF-CB-CTT, is based on lexicographic max-min fairness, to which we will refer to as max-min fairness for brevity. This fairness notion often appears in the context of network bandwidth allocation (see for example [BG92, SB08]). Max-min fairness is a purely qualitative measure of fairness, i. e., given two allocations, max-min fairness tells us which of the two is better, but not by how much. Our second problem formulation, JFI-CB-CTT, is based on Jain’s fairness index proposed by [JCH84]. This fairness measure is used in the famous AIMD algorithm by [CJ89] used in TCP Congestion Avoidance. In contrast to max-min fairness, it conveniently represents the inequality of a resource allocation as a number between zero and one.

In order to solve the MMF-CB-CTT problem we propose MAXMINFAIR_SA, an optimization algorithm based on simulated annealing (SA). Due to the mild requirements of SA on the problem structure, the proposed algorithm can easily be tailored to other max-min fair optimisation problems. A delicate part of the algorithm is the energy difference function, which quantifies how much worse one solution is compared to another solution – a piece of information we do not get directly from max-min fairness. We propose three different energy difference functions and evaluate their impact on the performance of MAXMINFAIR_SA on the 21 standard instances from track three of the ITC2007 competition ([DGS12]). Our experiments indicate, that the known best solutions with respect to the CB-CTT model are quite fair in the max-min sense, but further improvements are possible for 15 out of 21 instances and often a considerable improvement of the worst-off

stakeholder is achieved.

The fairness conditions imposed by max-min fairness are rather strict in the sense that there is no tradeoff between fairness and the total amount of penalty. In practice however, it may be desirable to pick a timetable from a number of solutions with varying tradeoffs between fairness and total penalty. Our proposed JFI-CB-CTT problem is a bi-criteria optimisation problem, which offers this option. We investigate the tradeoffs between fairness and total penalty for the six standard instances from [DGS12] whose known best solutions have the highest total penalty compared to the other instances. Our motivation for this choice of instances is simply that if the total penalty of a timetable is very small, then there is not much gain for anyone in distributing the penalty in a fair way. Our conclusion regarding this approach is that, although the known best solutions for the six instances are already quite fair, we can improve the fairness further at the cost of only a small increase in total penalty. For a theoretical treatment of the price of fairness on so-called convex utility sets with respect to proportional fairness and max-min fairness, see the recent work by [BFT11].

The remainder of this work is organized as follows. In Section 2, we will provide a brief review of the curriculum-based course timetabling (CB-CTT) problem model as well as max-min fairness and Jain’s fairness index. In Section 3 we will propose MMF-CB-CTT and JFI-CB-CTT, two fair variants of the CB-CTT problem formulation, and in Section 4, we will introduce the SA-based optimisation algorithm MAXMINFAIR_SA for solving max-min fair allocation problems. Section 5 is dedicated to our experimental evaluation of the fairness of the known best solutions to 21 standard instances from the website by [DGS12] with respect to max-min fairness and Jain’s fairness index, and the performance of the MAXMINFAIR_SA algorithm.

2 Preliminaries

In this section, we provide a brief review of the curriculum-based course timetabling problem formulation as well as relevant definitions concerning max-min fairness and Jain’s fairness index.

2.1 Curriculum-based Course Timetabling Problems

Curriculum-based Course Timetabling (CB-CTT) is a particular variant of the UCTP. It has been proposed in the course of the second international timetabling competition in 2007 (see [MSP⁺10]), and has since then emerged as one of the de-facto standard problem formulations in the timetabling community. The central entities in the CB-CTT formulation are the *curricula*, which are sets of lectures that must not be taught simultaneously. Both problem formulations proposed in the next section are based on the CB-CTT model.

CB-CTT problems are NP-hard and a lot of effort has been devoted to the development of exact and heuristic methods which provide high quality solutions within reasonable time. A wide range of techniques has been employed for solving CB-CTT instances including but not limited to approaches based on Max-SAT ([AAN10]), mathematical programming ([LL10, BMPR11]), local search ([DS06, LH10]), evolutionary computation ([ABM07]) as well as hybrid approaches ([BDS12, Mül09]). There has been a lot of progress in terms of the achieved solution quality in the

recent years. Interestingly, there seems to be no approach which is superior to the other approaches on most ITC2007 instances (see the website [DGS12] for current results).

A CB-CTT instance consists of a set of rooms, a set of courses, a set of curricula, a set of teachers and a set of days. Each day is divided into a fixed number of timeslots, a pair composed of a day and a timeslot is called a *period*. A period in conjunction with a room is called a *resource*. Each course consists of a number of lectures, i.e. a number of events to be scheduled, is taught by some teacher and has a number of students attending it. Each *curriculum* is a set of courses. For each room, we are provided with the maximum number of students it can accommodate and for each course we are given a list of periods in which it cannot be taught. A solution to a CB-CTT instance is a *timetable*, i.e. an assignment of courses to resources. The quality of a timetable is determined according to four hard and four soft constraints (see [DGMS07]).

The hard constraints are the following:

- H1 All lectures need to be scheduled and no two lectures of the same course may be assigned to the same period.
- H2 No two lectures may be assigned to the same resource.
- H3 Two courses in the same curriculum or taught by the same teacher must be assigned to different periods.
- H4 A lecture can only be scheduled in a period that is not marked unavailable for the corresponding course.

A timetable that satisfies all hard constraints is called *feasible*.

The CB-CTT formulation features the following soft constraints:

- S1 *RoomCapacity*: Each lecture should be assigned to a room of sufficient size.
- S2 *MinWorkingDays*: The lectures of each course should be distributed over a certain minimum number of days.
- S3 *IsolatedLectures*: For each curriculum, all lectures associated to the curriculum should be scheduled in adjacent timeslots.
- S4 *RoomStability*: The lectures of each course should be assigned to the same room.

Each violation of one of the soft constraints results in a “penalty” for the timetable. The CTT objective function aggregates individual penalties by taking their weighted sum. Detailed descriptions of how hard and soft constraints are evaluated and how much penalty is applied for a particular soft constraint violation can be found in the report by [DGMS07]. Given a CB-CTT instance I , the task is to find a feasible timetable such the aggregated penalty is minimal.

2.2 Fairness in Resource Allocation

Fairness issues typically arise when scarce resources are allocated to a number of stakeholders with demands. Fair resource allocation has received much attention in economic theory (see for example [FS06]), but also occurs in a wide range of applications in computer science including bandwidth allocation in networks ([BG92]) and task scheduling ([RDK05]). In many optimization problems related to resource allocation, the goal is to maximize the total amount of resources allocated to the stakeholders. Fairness in this context means that the distribution of resources over the stakeholders is important and that certain resource distributions are preferred over others.

Consider a resource allocation problem with n stakeholders resources. Each resource allocation (admissible solution) induces an allocation vector $X = (x_1, \dots, x_n)$, where each item x_i , $1 \leq i \leq n$, corresponds to the amount of resources allocated to stakeholder i . Typically, a preference for certain resource distributions is implicitly or explicitly contained in the objective function. For example, the task can be to find allocations maximizing the sum of the individual allocations, the mean allocation, the root mean square (RMS), the smallest allocation, and so forth (see [Ogr10, SK08]). When the fairness of an allocation is important we may be interested in improving the outcome for the worst-off stakeholders or generally try to allocate resources equally among the stakeholders. Max-min fairness is a notion of fairness that favours better outcomes for the worst-off stakeholders. It has received attention in the area of network engineering, in particular in the context of flow control [BFCYZ02, KRT01, SB08, ZLCJ12]. Various inequality measures have been proposed such as the Gini index proposed by [Gin21] and Jain's fairness index proposed by [JCH84]. Generally, a highly unequal distribution of resources is considered unfair. Our evaluation of fairness in academic course timetabling focuses on max-min fairness and Jain's fairness index.

Our evaluation of fairness in academic course timetabling focuses on the two fairness criteria max-min fairness and Jain's fairness index.

Max-min Fairness. Max-min fairness can be stated as iterated application of Rawls's Second Principle of Justice by [Raw99]:

“Social and economic inequalities are to be arranged so that they are to be of greatest benefit to the least-advantaged members of society.” (the Difference Principle)

Once the status of the least-advantaged members has been determined according to the difference principle, it can be applied again to everyone except the least-advantaged group in order to maximize the utility (in the economic sense) for the second least-advantaged members, and so on. The resulting utility assignment is called max-min fair. A max-min fair utility assignment implies that each stakeholder can maximize his/her utility as long as it is not at the expense of another stakeholder who is worse off. Thus, a max-min fair allocation enforces an efficient resource usage to some extent. A max-min fair resource allocation is Pareto-optimal.

In order to define max-min fairness more formally, we introduce some notation. Let X be an allocation vector. We generally assume that each entry of X is a nonnegative real number. By \tilde{X} we denote the vector containing the entries of X arranged in nondecreasing order. Similarly, let \bar{X} be a vector containing the entries of X in nonincreasing order. For allocation vectors X and Y we write $X \preceq_{mm} Y$ if X is at least as good as Y in the max-min sense. For maximization problems

such as bandwidth allocation this is the case iff $\vec{Y} \preceq_{lex} \vec{X}$, where \preceq_{lex} is the usual lexicographic comparison. For minimization problems such as the fair course timetabling problems proposed in Section 3, $X \preceq_{mm} Y$ iff $\tilde{X} \preceq_{lex} \tilde{Y}$. Let s be a solution to an instance I of an optimization problem and X be the allocation vector induced by s . Then s is called max-min fair, if for any other solution s' to I we have $X \preceq_{mm} Y$, where Y is the allocation vector induced by s' . Since the allocations are sorted, max-min fairness does not discriminate between stakeholders, but only between the amounts of resources assigned to them.

A weaker version of max-min fairness results if Rawl's Second Principle of Justice is not applied iteratively, but just once. This means that we are concerned with choosing the best possible outcome for the worst-off stakeholder. In the literature, related optimization problems are referred to as bottleneck optimization problems ([EF70, PZ11]). Note, that in contrast to max-min fairness, an optimal solution to a bottleneck optimisation problem is not necessarily Pareto-optimal. In the context of practical academic timetabling, the use of bottleneck optimization is hard to justify: each stakeholder is guaranteed to be at least as well off as the worst-off stakeholder, but no further improvement is considered.

Jain's Fairness Index. Jain's fairness index is an inequality measure proposed by Jain in [JCH84]. It is the crucial fairness measure that is used in the famous AIMD algorithm by [CJ89] used in TCP Congestion Avoidance. The fairness index $J(X)$ of an allocation vector X is defined as follows:

$$J(X) = \frac{\left(\sum_{1 \leq i \leq n} x_i \right)^2}{n \cdot \sum_{1 \leq i \leq n} x_i^2} . \quad (1)$$

It has several useful properties like population size independence, scale and metric independence, it is bounded between 0 and 1, and it has an intuitive interpretation. In particular $J(X) = 1$ means that X is a completely fair allocation, i. e., the allocation is fair for every stakeholder, and if $J(X) = 1/n$ then all resources are occupied by a single stakeholder. Furthermore, if $J(X) = x\%$ then the allocation X is fair for x percent of the stakeholders.

3 Fairness in Academic Course Timetabling

Course timetabling problems fit quite well in the framework of fair resource allocation problems described in the previous section: A timetable is an allocation of resources (rooms, timeslots) to lectures. In this section, we will define two fair versions of the CB-CTT problem formulation proposed by [MSP⁺10]. The first one, MMF-CB-CTT, is based on max-min fairness. Since max-min fairness enforces fairness as well as efficiency (maximum utility) at least to some extent, it is not a suitable concept for exploring the tradeoff between fairness and efficiency. Thus, we propose a second fair variant of CB-CTT called JFI-CB-CTT that is based on Jain's fairness index.

In order to use the fairness measures mentioned in the previous section, we need determine an allocation vector from a timetable. The central entities in the CB-CTT problem formulation

are the curricula. Therefore, in this work, we are interested in a fair distribution of penalty over the curricula. Depending on the application, a different set of stakeholders can be picked, but conceptually this does not change much. Let I be a CB-CTT instance with curricula c_1, c_2, \dots, c_k and let f_c be the CB-CTT objective function proposed by [DGMS07], which evaluates (S1)-(S4) restricted to curriculum c . This means f_c determines soft constraint violations only for the courses in curriculum c . For a timetable τ the corresponding allocation vector is given by the allocation function

$$A(\tau) = (f_{c_1}(\tau), f_{c_2}(\tau), \dots, f_{c_k}(\tau)) \quad . \quad (2)$$

Definition 1 (MMF-CB-CTT). *Given a CB-CTT instance I , the task is to find a feasible timetable τ such that $A(\tau)$ is max-min fair.*

If a feasible timetable has max-min fair allocation vector, then any curriculum c could receive less penalty only at the expense of other curricula which receive more penalty than c . This is due to the Pareto-optimality of a max-min fair allocation.

In order to explore the tradeoff between efficient and fair resource allocation in the context of the CB-CTT model, we propose another fair variant called JFI-CB-CTT that is based on Jain's fairness index proposed by [JCH84]. In order to get meaningful results from the fairness index however, we need a different allocation function. Consider an allocation X that allocates all penalty to a single curriculum while the remaining $k - 1$ curricula receive no penalty. Then $J(X) = 1/k$, which means that only one curriculum is happy with the allocation (see [JCH84]). In our situation however, the opposite is the case: $k - 1$ curricula are happy since they receive no penalty at all. The following allocation function shifts the penalty values such that the corresponding fairness index in the situation described above becomes $(k - 1)/k$, which is in much better agreement with our intuition:

$$A'(\tau) = (f_{\max} - f_{c_1}(\tau), f_{\max} - f_{c_2}(\tau), \dots, f_{\max} - f_{c_k}(\tau)) \quad , \quad (3)$$

with

$$f_{\max} = \max_{1 \leq i \leq k} \{f_{c_i}(\tau)\} \quad .$$

Definition 2 (JFI-CB-CTT). *Given a CB-CTT instance I , the task is to find the set of feasible solutions which are Pareto-optimal with respect to the two objectives of the objective function*

$$F(\tau) = (f(\tau), 1 - J(A'(\tau))) \quad , \quad (4)$$

where f is the CB-CTT objective function from [DGMS07] and J is defined in Eq. (1).

By a similar procedure, other classes of timetabling problem such as post-enrollment course timetabling, exam timetabling and nurse rostering can be turned into fair optimization problems. For example, for post-enrollment course timetabling, the central entities of interest are the individual students. Therefore, the goal were to achieve a fair distribution of penalty over all students. Once an appropriate allocation function has been defined, we immediately get the corresponding fair optimization problems.

Our proposed problem formulations are concerned with balancing the interests between stakeholders, who are in our case the students. In practice however, there are often several groups of

stakeholders with possibly conflicting interest, for example students, lecturers and administration. possibilities for extending the problem formulations above to include multiple stakeholders. For example, a multi-objective optimization approach may be considered, where each objective captures the fairness with respect to a particular stakeholder. When using inequality measures like Jain’s fairness index for different groups of stakeholders, the inequality values can be aggregated, for instance using a weighted-sum or ordered weighted averaging [Yag88] approach. Furthermore, max-min fairness or a suitable inequality measure can be applied to the different objectives to balance the interests of the different groups of stakeholders.

4 Simulated Annealing for Max-Min Fair Course Timetabling

Simulated Annealing (SA) is a popular local search method proposed by [KGV83], which works surprisingly well on many problem domains. SA has been applied successfully to timetabling problems by [Kos04] and [TD96]. Some of the currently known best solutions to CB-CTT instances from the ITC2007 competition were discovered by simulated annealing-based methods according to the website by [DGS12]. Our SA for max-min fair optimization problems shown in Algorithm 1 below (algorithm MAXMINFAIR_SA) is conceptually very similar to the original algorithm. The SA algorithm generates a new candidate solution according to some neighborhood exploration method, and replaces the current solution with a certain probability depending on i) the quality difference between the two solutions and ii) the current temperature. Since max-min fairness only tells us which of two given solutions is better, but not how much better, the main challenge in tailoring SA to max-min fair optimization problems is to find a suitable energy difference function, which quantifies the difference in quality between two candidate solutions. In the following, we propose three different energy difference measures for max-min fair optimization and provide details on the acceptance criterion, the cooling schedule, and the neighborhood exploration method chosen for the experimental evaluation of MAXMINFAIR_SA in the next section.

Algorithm 1: MAXMINFAIR_SA

input : s_{cur} : feasible timetable, ϑ_{max} : initial temperature, ϑ_{min} : final temperature, timeout
output: s_{best} : Best feasible timetable found so far

$s_{\text{best}} \leftarrow s_{\text{cur}}$

$\vartheta \leftarrow \vartheta_{\text{max}}$

while *timeout not hit* **do**

$s_{\text{next}} \leftarrow \text{neighbor}(s_{\text{cur}})$

if $P_{\text{accept}} \geq \text{random}()$ **then** $s_{\text{cur}} \leftarrow s_{\text{next}}$

if $A(s_{\text{cur}}) \preceq_{\text{mm}} A(s_{\text{best}})$ **then** $s_{\text{best}} \leftarrow s_{\text{cur}}$

$\vartheta \leftarrow \text{next_temperature}(\vartheta)$

end

return s_{best}

Acceptance Criterion. Similar to the original SA algorithm proposed by [KGV83], algorithm MAXMIN-FAIR-SA accepts an improved or equally good solution s_{next} with probability 1. If s_{next} is worse than s_{cur} then the acceptance probability depends on the current temperature level ϑ and the energy difference ΔE . The energy difference measures the difference in quality of the allocation induced by s_{next} compared to the allocation induced by the current solution s_{cur} . The acceptance probability P_{accept} is defined as:

$$P_{\text{accept}} = \begin{cases} 1 & \text{if } s_{\text{next}} \preceq_{mm} s_{\text{cur}} \\ \exp\left(-\frac{\Delta E(X, Y)}{\vartheta}\right) & \text{otherwise,} \end{cases}$$

where $X = A(s_{\text{cur}})$ and $Y = A(s_{\text{next}})$. In order to fit max-min fairness into the SA algorithm, we propose three energy difference measures: ΔE_{lex} , ΔE_{cw} , and ΔE_{ps} . ΔE_{lex} derives the energy difference from a lexicographic comparison, ΔE_{cw} from the component-wise ratios of the sorted allocation vectors and ΔE_{ps} from the ratios of the partial sums of the sorted allocation vectors. Our experiments presented in the next section indicate that the choice of the energy difference measure has a clear impact on the performance of MAXMINFAIR-SA and is thus a critical design choice.

For an allocation vector X , let \tilde{X}_i denote the i th entry after sorting the entries of X in nonincreasing order. The energy difference ΔE_{lex} of two allocation vectors X and Y of length n is defined as follows:

$$\Delta E_{\text{lex}}(X, Y) = 1 - \frac{1}{n} \cdot \left(\min_{1 \leq i \leq n} \left\{ i \mid \tilde{Y}_i > \tilde{X}_i \right\} - 1 \right). \quad (5)$$

ΔE_{lex} determines the energy difference between X and Y from the smallest index that determines $X \preceq_{mm} Y$. Thus, sorted allocation vectors which differ at the most significant indices have a higher energy difference than those differing at later indices. In particular, ΔE_{lex} evaluates to 1 if the minimum is 1, and it evaluates to $1/n$ if the minimum is n .

The energy difference measure ΔE_{lex} considers the earliest index at which two sorted allocation vectors differ but not how much the entries differ. We additionally propose the two energy difference measures ΔE_{cs} and ΔE_{ps} which take this information into account. These energy difference measures are derived from the definitions of approximation ratios for max-min fair allocation problems given by [KRT01]. An approximation ratio is a measure for how much worse the quality of a solution is relative to a possibly unknown optimal solution. In our case, we are interested in how much worse one given allocation is relative to another given allocation. We need to introduce some modifications of the definitions by [KRT01] since we are dealing with a minimization problem.

Note that due to (2), an allocation vector does not contain any positive entries. Let $\mu_{X, Y}$ be the smallest value of the two allocation vectors X and Y offset by a parameter $\delta > 0$, i. e.,

$$\mu_{X, Y} = \max\{\tilde{X}_1, \tilde{Y}_1\} + \delta. \quad (6)$$

The offset δ is introduced in order to avoid divisions by zero when taking ratios of penalty values.

The component-wise energy difference ΔE_{cw} of allocation vectors X and Y is defined as follows:

$$\Delta E_{\text{cw}}(X, Y) = \max_{1 \leq i \leq n} \left\{ \frac{\mu_{X, Y} - \tilde{Y}_i}{\mu_{X, Y} - \tilde{X}_i} \right\} - 1 \quad (7)$$

Since all entries are subtracted from $\mu_{X,Y}$, the ratios of the most significant entries with respect to \preceq_{mm} tend to dominate the value of ΔE_{cw} . Consider for example the situation that Y is much less fair than X , say, $\max\{\vec{X}_1, \vec{Y}_1\}$ occurs more often in X than in Y . Then for a small offset δ the energy difference $\Delta E_{cw}(X, Y)$ becomes large. On the other hand, if X is nearly as fair as Y then the ratios are all close to one and thus $\Delta E_{cw}(X, Y)$ is close to zero.

The third proposed energy difference measure ΔE_{ps} is based on the ratios of the partial sums $\sigma_i(X)$ of the sorted allocation vectors.

$$\sigma_i(X) = \sum_{1 \leq j \leq i} \tilde{X}_j .$$

The intention of using partial sums of the sorted allocations is to give the stakeholders receiving the most penalty more influence on the resulting energy difference compared to ΔE_{cw} . The energy difference ΔE_{ps} is defined as:

$$\Delta E_{ps}(X, Y) = \max_{1 \leq i \leq n} \left\{ \frac{i \cdot \mu_{X,Y} - \sigma_i(\vec{Y})}{i \cdot \mu_{X,Y} - \sigma_i(\vec{X})} \right\} - 1 . \quad (8)$$

Cooling Schedule. In algorithm MAXMINFAIR-SA, the function `next_temperature` updates the current temperature level ϑ according to the cooling schedule. We use a standard geometric cooling schedule

$$\vartheta = \alpha^t \cdot \vartheta_{max} ,$$

where α is the cooling rate and t is the elapsed time. Geometric cooling schedules decrease the temperature level exponentially over time. It is a popular class of cooling schedules which is widely used in practice and works well in many problem domains including timetabling problems [LA87, KAJ94, TD98]. Geometric cooling was chosen due to its simplicity, since the main focus of our evaluation in Section 5 is the performance impact of the different energy difference functions. We have made a slight adjustment to the specification of the geometric cooling schedule in order to make the behavior more consistent for different timeouts. Instead of specifying the cooling rate α , we determine α from ϑ_{max} , the desired minimum temperature ϑ_{min} and the timeout according to:

$$\alpha = \left(\frac{\vartheta_{min}}{\vartheta_{max}} \right)^{\frac{1}{\text{timeout}}} . \quad (9)$$

Thus, at the beginning ($t = 0$) the temperature level is ϑ_{max} and when the timeout is reached ($t = \text{timeout}$), the temperature level becomes ϑ_{min} . We chose to set a timeout rather than a maximum number of iterations since this setting is compliant with the ITC2007 competition conditions, which are a widely accepted standard for comparing results.

Neighborhood. In our max-min fair SA implementation, the function `neighbor` picks at random a neighbor in the Kempe-neighborhood of s_{cur} . The Kempe-neighborhood is the set of all timetables which can be reached by performing a single Kempe-move such that the number of lectures per period do not exceed the number of available rooms. The Kempe-move is a well-known and widely

Table 1: Fairness of the known best timetables from [DGS12] for the ITC2007 CB-CTT instances.

Instance	Curricula	$f(s_{best})$	$J(A'(s_{best}))$	$\vec{A}(s_{best})$
comp01	14	5	0.8571	$5^2, 0^{12}$
comp02	70	24	0.9515	$4, 2^{10}, 0^{59}$
comp03	68	66	0.9114	$13, 10^3, 9, 7^2, 6^4, 5^{13}, 4, 2^6, 0^{37}$
comp04	57	35	0.8964	$7, 6^3, 5^4, 4^2, 2, 0^{46}$
comp05	139	291	0.8277	$41^2, 36^7, 35^5, 32^5, 31^6, 30^9, 28, 27^7, 26^2, 25^{14}, \dots, 2, 0^3$
comp06	70	27	0.9657	$12, 7^2, 5^4, 2^3, 0^{60}$
comp07	77	6	0.9870	$6, 0^{76}$
comp08	61	37	0.9020	$7, 6^3, 5^4, 4^2, 2^2, 0^{49}$
comp09	75	96	0.8047	$10^5, 9, 7^{10}, 6^6, 5^{10}, 4, 2, 0^{41}$
comp10	67	4	0.9701	$2^2, 0^{65}$
comp11	13	0	—	0^{13}
comp12	150	300	0.9128	$45, 30^{14}, 28, 27^2, 26^5, 25^{19}, 22^4, 21^6, 20^8, 19, \dots, 2^2, 0^3$
comp13	66	59	0.8830	$8, 7, 6^5, 5^7, 4^2, 2^3, 0^{47}$
comp14	60	51	0.9023	$8^4, 7, 5^2, 2^6, 0^{47}$
comp15	68	66	0.8495	$10^3, 9^3, 7, 6^4, 5^{13}, 4, 2^7, 0^{36}$
comp16	71	18	0.9176	$7^2, 5^7, 4, 0^{61}$
comp17	70	56	0.9248	$10^2, 6^3, 5^9, 2^4, 0^{52}$
comp18	52	62	0.9009	$17, 15, 14, 13, 11, 10, 9^2, 5^{19}, 2^2, 0^{23}$
comp19	66	57	0.9612	$13, 7, 6^4, 5^2, 4, 2^7, 0^{50}$
comp20	78	4	0.9744	$2^2, 0^{76}$
comp21	78	76	0.8838	$12, 11, 10^4, 9, 7^4, 6^4, 5^{12}, 4, 2^3, 1^2, 0^{45}$

used operation for swapping events in a timetable [BEM⁺10, LH10, MBHS02, TD98, TBM07]. A prominent feature of the Kempe-neighborhood is that it contains only moves that preserve the feasibility of a timetable. Since the algorithm MAXMINFAIR-SA only uses moves from this neighborhood the output is guaranteed to be a feasible timetable. In the future, more advanced neighborhood exploration methods similar to the approaches in [DS06, LH10] could be used, which may well lead to an improved overall performance of MAXMINFAIR-SA.

5 Evaluation

In this section, we will first address the question how fair or unfair the known best timetables for the ITC2007 CB-CTT instances are with respect to Jain’s fairness index and max-min fairness. Table 1 shows our measurements for all instances comp01, comp02, . . . , comp21 from the ITC2007 competition (see [DGS12] for instance data). Please note that the known best timetables were not created with fairness in mind, but the objective was to create timetables with minimal total penalty. In Table 1, s_{best} refers to the known best solution for each instance. A and A' refer to the allocation functions given in (2) and (3), respectively. The data indicates that the timetables with a low total

Table 2: The performance of MAXMINFAIR_SA with $\Delta E = \Delta E_{cw}$ for different values of δ .

δ	10^0	10^{-2}	10^{-3}	10^{-6}
10^0	–	02	02, 05	02
10^{-2}	10, 19, 20	–	09	19
10^{-3}	01, 10, 19, 20	–	–	03, 19
10^{-6}	01, 10, 20	–	–	–

penalty are also rather fair. This can be explained by the fact that these timetables do not have a large amount of penalty to distribute over the curricula. Thus, most curricula receive little or no penalty and consequently, the distribution is fair for most curricula. We will show in Section 5.2 however, that for timetables with a comparatively large total penalty there is still some room for improvement concerning fairness.

The rightmost column of Table 1 contains the sorted allocation vectors of the best solutions. For a more convenient presentation, all entries of the sorted allocation vectors are multiplied by -1. The exponents denote how often a certain number occurs. For example, the sorted allocation vector $(-5, -5, 0, 0, 0)$ would be represented as $5^2, 0^3$. The sum of the values of an allocation vector is generally much larger than the total penalty shown in the second column. The reason for this is that the penalty assigned to a course is counted for each curriculum the course belongs to. With a few exceptions the general theme seems to be that the penalty is assigned to only a few curricula while a majority of curricula receives no penalty. In the next section we will show that the situation for the curricula which receive the most penalty can be improved with max-min fair optimization for 15 out of 21 instances.

5.1 Max-Min Fair Optimization

In Section 4, we presented algorithm MAXMINFAIR_SA for solving max-min fair minimization problems. A crucial part of this algorithm is the energy difference measure which determines how much worse a given solution is compared to another solution, i.e. the energy difference of the solutions. We evaluate the impact of the three energy difference measures (5), (7) and (8) on the performance of MAXMINFAIR_SA.

Our test setup was the following: For each energy difference function we independently performed 50 runs with MAXMINFAIR_SA. The temperature levels were determined experimentally, we set $\vartheta_{max} = 5$ and $\vartheta_{min} = 0.01$; the cooling rate α was set according to (9). In order to establish consistent experimental conditions for fair optimization, we used a timeout, which was determined according to the publicly available ITC2007 benchmark executable. On our machines (i7 CPUs running at 3.4GHz, 8GB RAM), the timeout was set to 192 seconds. The MAXMINFAIR_SA algorithm was executed on a single core. We generated feasible initial timetables for MAXMINFAIR_SA as a preprocess using sequential heuristics proposed by [BMM⁺07]. The soft constraint violations were not considered at this stage. Since the preprocess was performed only once per instance (not per run), it is not counted in the timeout. However, the time it took was negligible compared to the timeout (less than 1 second per instance).

Table 3: The performance of MAXMINFAIR_SA with energy difference measures ΔE_{lex} , ΔE_{ps} and ΔE_{cw} .

ΔE	ΔE_{lex}	ΔE_{ps}	ΔE_{cw}
ΔE_{lex}	–	–	–
ΔE_{ps}	all except 01, 06, 08, 11, 17	–	18
ΔE_{cw}	all except 11	06, 07, 08, 17, 21	–

Table 2 shows the impact of the parameter δ on the performance of MAXMINFAIR_SA with energy difference measure ΔE_{cw} . For each pair of values we performed the one-sided Wilcoxon Rank-Sum test with a significance level of 0.01. The data indicates that for best performance, δ should be small, but not too small. For $\delta = 1$, MAXMINFAIR_SA beats the other shown configurations on instance comp02 but performs worse than the other configurations on instances comp10 and comp20. For $\delta = 10^{-6}$ the overall performance is better than for $\delta = 1$, but worse than for the other configurations. With $\delta = 10^{-2}$ and $\delta = 10^{-3}$, MAXMINFAIR_SA shows the best relative performance. Thus, for our further evaluation we set $\delta = 10^{-3}$.

Table 3 shows the relative performance of Algorithm MAXMINFAIR_SA for the proposed energy difference measures (5), (7) and (8). The table shows for any choice of two energy difference measures i and j , for which instances MAXMINFAIR_SA with measure i performs significantly better than MAXMINFAIR_SA using measure j . Again, we used the Wilcoxon Rank-Sum test with a significance level of 1 percent. The data shows that ΔE_{cw} is the best choice among the three alternatives, since it is a better choice than ΔE_{lex} on all instances except comp11 and a better choice than ΔE_{ps} on five out of 21 instances. However, although ΔE_{cw} shows significantly better performance than the other energy difference measures, it did not necessarily produce the best timetables on all instances. For the instances comp03, comp15, comp05 and comp12 for example, the best solution found with $\Delta E = \Delta E_{ps}$ was better than with $\Delta E = \Delta E_{cw}$.

The data in Table 4 shows a comparison of the sorted allocation vectors of the known best solutions from the CB-CTT website by [DGS12] with the best solutions found by the 50 runs of MAXMINFAIR_SA with $\Delta E = \Delta E_{cw}$. First of all, for instances comp01 and comp11, the allocation vectors of the best existing solutions and the best solution found by MAXMINFAIR_SA are identical. This means that MAXMINFAIR_SA finds reasonably good solutions despite the certainly more complex fitness landscape due to max-min fair optimization. We can also observe that the maximum penalty any curriculum receives is significantly less for most instances and the penalty is more evenly distributed across the curricula. This means that although max-min fair timetables may have a higher total penalty, they might be more attractive from the students' perspective, since in the first place each student notices an unfortunate arrangement of his/her timetable, which is tied to the curriculum. Furthermore, we can observe that if the total penalty of a known best solution is rather low, then it is also good with respect to max-min fairness. For several instances in this category, (comp01, comp04, comp07, comp10 and comp20), the solution found by MAXMINFAIR_SA is not as good as the known best solution with respect to max-min fairness. We can conclude that if there is not much penalty to distribute between the stakeholders, it is not necessary to enforce a fair distribution of penalty.

Table 4: Comparison of the sorted allocation vectors of the known best solutions from the website by [DGS12] with the allocation vectors found by MAXMINFAIR_SA with respect to max-min fairness.

Instance	Known best solution	MAXMINFAIR_SA ($\Delta E = \Delta E_{cw}$)
comp01	$5^2, 0^{12}$	$5^2, 0^{12}$
comp02	$4, 2^{10}, 0^{59}$	$4^2, 2^{31}, 1^7, 0^{30}$
comp03	$13, 10^3, 9, 7^2, 6^4, 5^{13}, 4, 2^6, 0^{37}$	$6^4, 4^{11}, 2^{22}, 1^3, 0^{28}$
comp04	$7, 6^3, 5^4, 4^2, 2, 0^{46}$	$6^4, 4^2, 2^4, 1, 0^{46}$
comp05	$41^2, 36^7, 35^5, 32^5, 31^6, 30^9, 28, \dots, 2, 0^3$	$19^2, 18^3, 17^3, 16^5, 15^2, 14^{15}, 13^5, \dots, 4^8, 3^3, 2$
comp06	$12, 7^2, 5^4, 2^3, 0^{60}$	$12, 4^2, 2^{30}, 1^{13}, 0^{24}$
comp07	$6, 0^{76}$	$6, 2^{23}, 1^{24}, 0^{29}$
comp08	$7, 6^3, 5^4, 4^2, 2^2, 0^{49}$	$6^4, 4^2, 2^7, 1^5, 0^{43}$
comp09	$10^5, 9, 7^{10}, 6^6, 5^{10}, 4, 2, 0^{41}$	$6^9, 4^{14}, 2^{17}, 0^{35}$
comp10	$2^2, 0^{65}$	$2^{19}, 1^6, 0^{42}$
comp11	0^{13}	0^{13}
comp12	$45, 30^{14}, 28, 27^2, 26^5, 25^{19}, 22^4, \dots, 2^2, 0^3$	$10^3, 9^6, 8^{31}, 7^7, 6^{43}, 5^2, 4^{36}, 3^2, 2^{16}, 1, 0^3$
comp13	$8, 7, 6^5, 5^7, 4^2, 2^3, 0^{47}$	$6^6, 4^4, 2^{13}, 1^6, 0^{37}$
comp14	$8^4, 7, 5^2, 2^6, 0^{47}$	$8^4, 4^2, 3, 2^{18}, 0^{35}$
comp15	$10^3, 9^3, 7, 6^4, 5^{13}, 4, 2^7, 0^{36}$	$6^4, 4^{11}, 2^{23}, 1^2, 0^{28}$
comp16	$7^2, 5^7, 4, 0^{61}$	$4^5, 2^{16}, 1^4, 0^{46}$
comp17	$10^2, 6^3, 5^9, 2^4, 0^{52}$	$10^2, 6^2, 4^7, 3, 2^{25}, 1^7, 0^{26}$
comp18	$17, 15, 14, 13, 11, 10, 9^2, 5^{19}, 2^2, 0^{23}$	$4^{20}, 2^{11}, 1^5, 0^{16}$
comp19	$13, 7, 6^4, 5^2, 4, 2^7, 0^{50}$	$6^4, 4^6, 2^{15}, 1^{14}, 0^{27}$
comp20	$2^2, 0^{76}$	$4^5, 3^3, 2^{31}, 1^7, 0^{32}$
comp21	$12, 11, 10^4, 9, 7^4, 6^4, 5^{12}, 4, 2^3, 1^2, 0^{45}$	$10, 6^4, 5, 4^{15}, 3, 2^{36}, 1^3, 0^{17}$

5.2 The Tradeoff Between Fairness and Efficiency

We proposed the JFI-CB-CTT problem formulation in Section 3, which allows us to investigate the tradeoff between fairness and efficiency which arises in course timetabling. We can observe in column 4 of Table 1 that for all of the best solutions from [DGS12] the fairness index (1) is greater than 0.8, i.e., the known best solutions are also fair for more than 80 percent of the curricula. In order to solve the corresponding JFI-CB-CTT instances, we use the multi-objective optimization algorithm AMOSA proposed by [BSMD08] that is based on simulated annealing like Algorithm MAXMINFAIR_SA. Since we do not expect from a general multi-objective optimization algorithm to produce solutions as good as the best CB-CTT solvers, we will consider the following scenario to explore the tradeoffs between fairness and efficiency: starting from the known best solution we examine how much increase in total penalty we have to tolerate in order to increase the fairness further. We will take as examples the six instances with the highest total amount of penalty, comp03, comp05, comp09, comp12, comp15 and comp21.

The temperature levels for the AMOSA algorithm were set to $\vartheta_{max} = 20$ and $\vartheta_{min} = 0.01$; α was set according to (9) with a timeout determined by the official ITC2007 benchmark. The plots in Figure 1 show the (Pareto-) non-dominated solutions found by AMOSA. The arrows point to the starting point, i.e. the best available solutions to the corresponding instances. For instances comp05 and comp21 solutions with a lower total cost than the the previously known best solutions were discovered by this approach. The plots show that the price for increasing the fairness is generally not very high – up to a certain level, which depends on the instance. In fact, for comp09 and comp21, the fairness index can be increased by 3.5 percent and 1.4 percent, respectively, without increasing the total penalty at all.

In Figure 1, the straight lines that go through the initial solutions show a possible tradeoff between fairness and efficiency: the slopes were determined such that a 1 percent increase in fairness yields a 1 percent increase in penalty. For the instances shown in Figure 1, the solutions remain close to the tradeoff lines up to a fairness of 94 to 97 percent, while a further increase in fairness demands a significant increase in total cost. For the instances comp05, comp09 and comp15, there are several solutions below the tradeoff lines. Picking any of the solutions below these lines would result in an increased fairness without an equally large increase in the amount of penalty. This means picking a fairer solution might well be an attractive option in a real-world academic timetabling context. For comp05 for example, the fairness of the formerly best known solution with a total penalty of 291 can be increased by 5.4 percent at 302 total penalty, which is a 3.8 percent increase.

In summary, improving the fairness of an efficient timetable as a post-processing step seems like a viable approach for practical decision making. Using a very efficient solution as a starting point means that we can benefit from the existing very good approaches to creating timetables with minimal total cost and provide improved fairness depending on the actual, instance-dependent tradeoff.

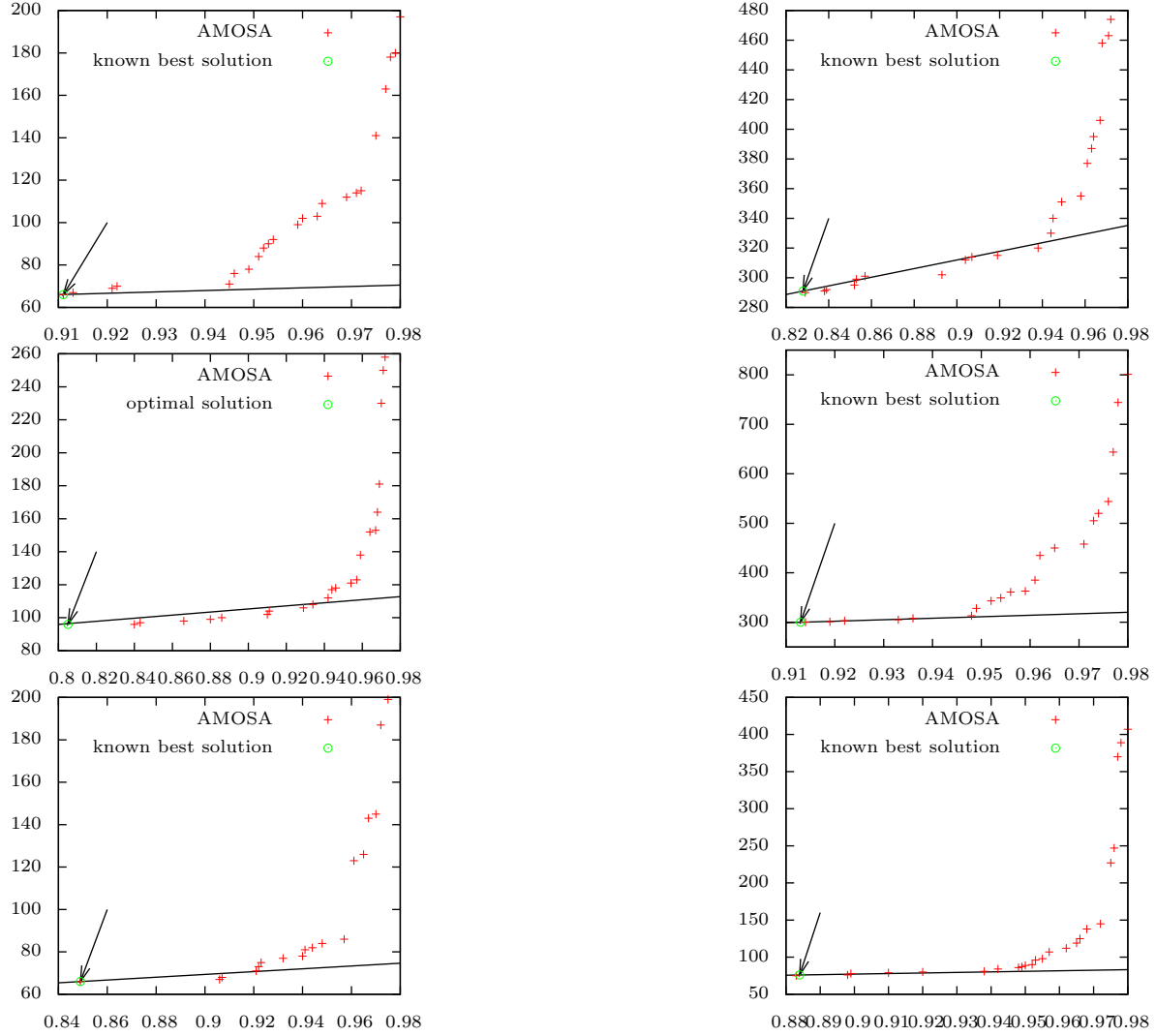


Figure 1: Non-dominated solutions found by the AMOSA algorithm for the JFI-CB-CTT versions of instances comp03, comp05, comp09, comp12, comp15 and comp21. All graphs show the fairness index on the horizontal axis and the amount of penalty on the vertical axis.

6 Conclusion

In this paper we introduced two new problem formulations for academic course timetabling based on the CB-CTT problem model from track three of the ITC2007, MMF-CB-CTT and JFI-CB-CTT. Both problem formulations are aimed at creating fair course timetables in the setting of a university but include different notions of fairness. Fairness in our setting means that the penalty assigned to a timetable is distributed in a fair way among the different curricula. The MMF-CB-CTT formulation aims at creating max-min fair course timetables while JFI-CB-CTT is a bi-objective problem formulation based on Jain’s fairness index. The motivation for the JFI-CB-CTT formulation is to explore the tradeoff between a fair penalty distribution and a low total penalty.

Furthermore, we proposed an optimization algorithm based on simulated annealing for solving MMF-CB-CTT problems. A critical part of the algorithm is concerned with measuring the energy difference between two timetables, i.e., how much worse a timetable is compared to another timetable with respect to max-min fairness. We evaluated the performance of the proposed algorithm for three different energy difference measures on the 21 CB-CTT benchmark instances. Our results show clearly that the algorithm performs best with ΔE_{cw} as energy difference measure.

Additionally, we investigated the fairness of the known best solutions of the 21 CB-CTT instances with respect to max-min fairness and Jain’s fairness index. These solutions were not created with fairness in mind, but our results show that all of the solutions have a fairness index greater than 0.8. This means they can be considered quite fair. Nevertheless, our results show that some improvements are possible with respect to both max-min fairness and Jain’s fairness index. The timetables produced by our proposed MAXMINFAIR_SA algorithms are better than the known best ones with respect to max-min fairness for 15 out of 21 instances. Our investigation of the tradeoff between fairness and the total amount of penalty using the JFI-CB-CTT problem formulation shows that the fairness of the known best timetables can be increased further with only a small increase of the total penalty.

References

- [AAN10] Roberto Asín Acha and Robert Nieuwenhuis. Curriculum-based course timetabling with SAT and MaxSAT. In *Proc. 8th Int. Conf. on the Practice and Theory of Automated Timetabling (PATAT)*, pages 42–56, 2010.
- [ABM07] Salwani Abdullah, Edmund K. Burke, and Barry McCollum. A hybrid evolutionary approach to the university course timetabling problem. In *IEEE Congress on Evolutionary Computation (CEC)*, pages 1764–1768, 2007. doi:10.1109/CEC.2007.4424686.
- [BDS12] Ruggero Bellio, Luca DiGasparo, and Andrea Schaerf. Design and statistical analysis of a hybrid local search algorithm for course timetabling. *Journal of Scheduling*, 15:49–61, 2012. doi:10.1007/s10951-011-0224-2.

- [BEM⁺10] Edmund K. Burke, Adam J. Eckersley, Barry McCollum, Sanja Petrovic, and Rong Qu. Hybrid variable neighbourhood approaches to university exam timetabling. *European Journal of Operational Research*, 206(1):46–53, 2010. doi:10.1016/j.ejor.2010.01.044.
- [BFCYZ02] Yair Bartal, Martin Farach-Colton, Shibu Yooseph, and Lisa Zhang. Fast, fair and frugal bandwidth allocation in ATM networks. *Algorithmica*, 33(3):272–286, 2002. doi:10.1007/s00453-001-0119-2.
- [BFT11] Dimitris Bertsimas, Vivek F. Farias, and Nikolaos Trichakis. The price of fairness. *Operations Research*, 59(1):17–31, February 2011. doi:10.1287/opre.1100.0865.
- [BG92] Dimitri P. Bertsekas and Robert Gallager. *Data Networks*. Prentice Hall, 2nd edition, 1992.
- [BMM⁺07] Edmund K. Burke, Barry McCollum, Amnon Meisels, Sanja Petrovic, and Rong Qu. A graph-based hyper-heuristic for educational timetabling problems. *European Journal of Operational Research*, 176(1):177–192, 2007. doi:10.1016/j.ejor.2005.08.012.
- [BMPR11] Edmund K. Burke, Jakub Mareček, Andrew J. Parkes, and Hana Rudová. A branch-and-cut procedure for the Udine Course Timetabling problem. *Annals of Operations Research*, 194(1):71–87, 2011. doi:10.1007/s10479-010-0828-5.
- [BSMD08] Sanghamitra Bandyopadhyay, Sriparna Saha, Ujjwal Maulik, and Kalyanmoy Deb. A simulated annealing-based multiobjective optimization algorithm: AMOSA. *IEEE Transactions on Evolutionary Computation*, 12:269–283, 2008. doi:10.1109/TEVC.2007.900837.
- [CJ89] Dah-Ming Chiu and Raj Jain. Analysis of the increase and decrease algorithms for congestion avoidance in computer networks. *Computer Networks and ISDN Systems*, 17:1–14, 1989. doi:10.1016/0169-7552(89)90019-6.
- [DGMS07] Luca Di Gaspero, Barry McCollum, and Andrea Schaerf. The second international timetabling competition (ITC-2007): Curriculum-based course timetabling (Track 3). Technical Report QUB/IEEE/Tech/ITC2007/CurriculumCTT/v1.0/1, School of Electronics, Electrical Engineering and Computer Science, Queens University, Belfast (UK), August 2007.
- [DGS12] Luca Di Gaspero and Andrea Schaerf. Curriculum-based course timetabling site, January 2012. <http://satt.diegm.uniud.it/ctt/>.
- [DS06] Luca Di Gaspero and Andrea Schaerf. Neighborhood portfolio approach for local search applied to timetabling problems. *Journal of Mathematical Modelling and Algorithms*, 5:65–89, 2006. doi:10.1007/s10852-005-9032-z.
- [EF70] Jack Edmonds and D.R. Fulkerson. Bottleneck extrema. *Journal of Combinatorial Theory*, 8(3):299–306, 1970. doi:10.1016/S0021-9800(70)80083-7.

- [FS06] Allan M. Feldman and Roberto Serrano. *Welfare economics and social choice theory*. Springer, New York, NY, 2nd edition, 2006. doi:10.1007/0-387-29368-X.
- [Gin21] Corrado Gini. Measurement of inequality of incomes. *The Economic Journal*, 31(121):124–126, January 1921. doi:10.2307/2223319.
- [JCH84] Rajendra K. Jain, Dah-Ming W. Chiu, and William R. Hawe. A quantitative measure of fairness and discrimination for resource allocation in shared computer systems. Technical Report DEC-TR-301, Digital Equipment Corporation, September 1984. arXiv:cs/9809099.
- [KAJ94] Christos Koulamas, Solomon Antony, and R. Jaen. A survey of simulated annealing applications to operations research problems. *Omega*, 22(1):41–56, 1994. doi:10.1016/0305-0483(94)90006-X.
- [KGV83] Scott Kirkpatrick, C. D. Gelatt, and M. P. Vecchi. Optimization by simulated annealing. *Science*, 220(4598):671–680, 1983. doi:10.1126/science.220.4598.671.
- [KMT98] Frank Kelly, A. Maulloo, and D. Tan. Rate control in communication networks: shadow prices, proportional fairness and stability. *Journal of the Operational Research Society*, 49, 1998.
- [Kos04] Philipp Kostuch. The university course timetabling problem with a three-phase approach. In *Proc. 5th Int. Conf. on the Practice and Theory of Automated Timetabling (PATAT)*, pages 109–125. Springer, 2004. doi:10.1007/11593577_7.
- [KRT01] Jon Kleinberg, Yuval Rabani, and Éva Tardos. Fairness in routing and load balancing. *Journal of Computer and System Sciences*, 63(1):2–20, 2001. doi:10.1006/jcss.2001.1752.
- [LA87] P. J. M. van Laarhoven and E. H. L. Aarts. *Simulated Annealing: Theory and Applications*. Kluwer Academic Publishers, 1987.
- [LH10] Zhipeng Lü and Jin-Kao Hao. Adaptive tabu search for course timetabling. *European Journal of Operational Research*, 200(1):235–244, 2010. doi:10.1016/j.ejor.2008.12.007.
- [LL10] Gerald Lach and Marco E. Lübbecke. Curriculum based course timetabling: new solutions to Udine benchmark instances. *Annals of Operations Research*, pages 1–18, 2010. doi:10.1007/s10479-010-0700-7.
- [MBHS02] Liam T. G. Merlot, Natashia Boland, Barry D. Hughes, and Peter J. Stuckey. A hybrid algorithm for the examination timetabling problem. In *Proc. 4th Int. Conf. on the Practice and Theory of Automated Timetabling (PATAT)*, pages 207–231. Springer, 2002. doi:10.1007/978-3-540-45157-0_14.

- [MSP⁺10] Barry McCollum, Andrea Schaerf, Ben Paechter, Paul McMullan, Rhyd Lewis, Andrew J. Parkes, Luca Di Gaspero, Rong Qu, and Edmund K. Burke. Setting the research agenda in automated timetabling: The second international timetabling competition. *INFORMS Journal on Computing*, 22:120–130, 2010. doi:10.1287/ijoc.1090.0320.
- [Mül09] Tomáš Müller. ITC2007 solver description: a hybrid approach. *Annals of Operations Research*, 172(1):429–446, 2009. doi:10.1007/s10479-009-0644-y.
- [Ogr10] Włodzimierz Ogryczak. Bicriteria models for fair and efficient resource allocation. In *Proceedings of the 2nd International Conference on Social Informatics (SocInfo)*, pages 140–159. Springer, 2010. doi:10.1007/978-3-642-16567-2_11.
- [OW04] Włodzimierz Ogryczak and Adam Wierzbicki. On multi-criteria approaches to bandwidth allocation. *Control and Cybernetics*, 33:427–448, 2004.
- [PZ11] Abraham P. Punnen and Ruonan Zhang. Quadratic bottleneck problems. *Naval Research Logistics (NRL)*, 58(2):153–164, 2011. doi:10.1002/nav.20446.
- [Raw99] John Rawls. *A Theory of Justice*. Belknap Press of Harvard University Press, revised edition, 1999.
- [RDK05] Thomas Repantis, Yannis Drougas, and Vana Kalogeraki. Adaptive resource management in Peer-to-Peer middleware. In *Proc. 19th IEEE Int. Parallel and Distributed Processing Symposium (IPDPS)*, 2005. doi:10.1109/IPDPS.2005.80.
- [SB08] Ronaldo M. Salles and Javier A. Barria. Lexicographic maximin optimisation for fair bandwidth allocation in computer networks. *European Journal of Operational Research*, 185(2):778–794, 2008. doi:10.1016/j.ejor.2006.12.047.
- [SF97] Amartya Sen and James E. Foster. *On economic inequality*. Clarendon Press ; Oxford University Press, Oxford : New York :, enl. ed., with a substantial annexe ”on economic inequality after a quarter century” / james foster and amartya kumar sen. edition, 1997.
- [SK08] M. J. Soomer and G. M. Koole. Fairness in the aircraft landing problem. In *Proceedings of the Anna Valicek Competition 2008*, 2008.
- [TBM07] Mauritsius Tuga, Regina Berretta, and Alexandre Mendes. A hybrid simulated annealing with Kempe chain neighborhood for the university timetabling problem. In *Proceedings of the 6th ACIS International Conference on Computer and Information Science (ACIS-ICIS)*, pages 400–405, 2007. doi:10.1109/ICIS.2007.25.
- [TD96] Jonathan Thompson and Kathryn A. Dowsland. General cooling schedules for a simulated annealing based timetabling system. In *Proc. 1st Int. Conf. on the Practice and Theory of Automated Timetabling (PATAT)*, pages 345–363, 1996. doi:10.1007/3-540-61794-9_70.

- [TD98] Jonathan M. Thompson and Kathryn A. Dowsland. A robust simulated annealing based examination timetabling system. *Computers & Operations Research*, 25(7-8):637–648, 1998. doi:10.1016/S0305-0548(97)00101-9.
- [Yag88] Ronald R. Yager. On ordered weighted averaging aggregation operators in multicriteria decisionmaking. *IEEE Transactions on Systems, Man and Cybernetics*, 18(1):183–190, January 1988. doi:10.1109/21.87068.
- [ZLCJ12] Liang Zhang, Wen Luo, Shigang Chen, and Ying Jian. End-to-end maxmin fairness in multihop wireless networks: Theory and protocol. *Journal of Parallel and Distributed Computing*, 72(3):462–474, 2012. doi:10.1016/j.jpdc.2011.11.012.